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Bond Returns**

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Abstract: The authors examine how the co-movement between daily stock and Treasury bond returns varies with stock market uncertainty. They use the lagged implied volatility from equity index options to provide an objective, observable, and dynamic measure of stock market uncertainty. The authors find that stock and bond returns tend to move substantially together during periods of lower stock market uncertainty. However, stock and bond returns tend to exhibit little relation or even a negative relation during periods of high stock market uncertainty. The authors' findings have implications for understanding joint cross-market price formation. Further, their findings imply that diversification benefits increase for portfolios of stocks and bonds during periods of high stock market uncertainty.

JEL classification: G11, G12, G14

Key words: stock and bond market return linkages, stock market uncertainty, time-varying volatility

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Stock market uncertainty and the relation between stock and bond returns

1 Introduction

It is well known that stock-market volatility exhibits substantial variation over time. Further, this variation in stock volatility is puzzling because it seems excessive based on the time-variation in the volatility of fundamental economic variables (e.g., Schwert, 1989; Haugen, Talmor, and Torous, 1991; Whitelaw, 1994; Campbell, Lettau, Malkiel, and Xu, 2001).

It is also well known that stock and bond returns are positively correlated. However, the unconditional correlation is small and a second body of research has tried to understand the co-movements between the stock and bond markets (e.g., Barsky, 1989; Fama and French, 1989; Shiller and Beltratti, 1992; Campbell and Ammer, 1993; and Fleming, Kirby, and Ostdiek, 1998).

In this paper, we are interested in the natural question of whether stock market uncertainty has a role in understanding stock and bond market co-movements. If the relative attractiveness of bonds versus stocks changes with stock market uncertainty, then the co-movement between stock and bond returns may vary with stock market uncertainty. Changes in return co-movements might be due to changing fundamentals (Campbell and Ammer, 1993), cross-market hedging (Fleming, Kirby, and Ostdiek, 1998), or pricing influences related to time-varying economic uncertainty and possible regime shifting (Veronesi, 1999 and 2001).

The idea that economic uncertainty and unobservable regime-shifting may be important in understanding return dynamics seems related to the notion of “flight-to-quality” that is often suggested in the popular press. The notion of flight-to-quality suggests that during times of increased stock uncertainty: (1) the price of U.S. Treasury bonds tends to increase, relative to stocks; and (2) the return co-movement between stocks and bonds becomes less positively correlated (or even negatively correlated). For example, an article in the Wall Street Journal on 11/4/97 (around the Asian financial crisis) speculates that the recent decoupling between the stock and bond markets may be due to the high stock volatility and uncertain economic times. Or, in the Wall Street Journal of 10/17/89, an article about the bond market states, “The sudden flight-to-quality that

triggered Friday’s explosive bond-market rally was reversed yesterday in a flight-from-quality rout. The setback, in which Treasury bond prices plummeted, reflected a rebound in the stock market and profit-taking.” This second article suggests the possibility of sustained periods of negative correlation between stock and bond returns.

To measure changes in perceived stock market risk or uncertainty, we use the implied volatility from equity index options. We use the lagged implied Volatility Index (VIX) from the Chicago Board Option’s Exchange to provide an objective, observable, and dynamic measure of stock market uncertainty. Recent studies find that the information in implied volatility provides the best volatility forecast and largely subsumes the volatility information from historical return shocks (e.g., Christensen and Prabhala, 1998; Fleming, 1998; and Blair, Poon, and Taylor, 2001).

Under the standard Black-Scholes assumptions, implied volatility should only reflect expected stock market volatility. However, the Black-Scholes implied volatility of equity index options has been shown to be biased high. Recent papers such as Coval and Shumway (2000), Poteshman (2000), and Bakshi and Kapadia (2001) present evidence that option prices may also contain a component that reflects the risk of stochastic volatility. If options are valuable as hedges against unanticipated increases in volatility, then option prices may be higher than expected under a Black-Scholes world of known volatility. If so, option prices would typically yield a Black-Scholes implied volatility that is higher than realized volatility, which could explain the above noted bias. In this sense, time-varying implied volatility may reflect both movements in the market’s expected volatility and movements in the market’s uncertainty about future volatility. Further, David and Veronesi (2000) present a regime-shifting model where implied volatility is higher in times investors are more uncertain about the state of earnings growth. For the purposes of this article, we lump these possible interpretation of implied volatility together and refer to movements in implied volatility as movements in ”stock market uncertainty”.¹

Understanding stock and bond market co-movements has important practical implications in several areas. First, asset allocation between stocks and bonds is one of the fundamental decisions that portfolio managers and individual investors must make. Second, understanding volatility

¹The CBOE’s Volatility Index is also commonly referred to as a market “Fear Index”.

linkages and conditional correlations also has a role in risk management and derivative valuation.

Our empirical investigation yields several noteworthy findings. First, we form subsets of returns by sorting on the lagged VIX level. Across these subsets, we find that the simple correlation between stock and bond returns varies negatively and substantially with the lagged VIX level. For the largest lagged VIX decile, the stock-bond correlation is even negative.

Second, we estimate a model that allows the relation between stocks and bonds to vary as a continuous function of the lagged VIX level. We find that the lagged VIX is a useful and reliable instrument in explaining variation in the relation between stock and bond returns. For example, when the lagged VIX is at its 5th percentile, the implied contemporaneous relation between 10-year bond and stock returns in our model is 0.352. In contrast, when the lagged VIX is at its 95th percentile, the implied contemporaneous relation between the 10-year bond and stock returns is 0.012. Even at a one-month lag, the VIX provides reliable information about the co-movement between stock and bond returns. Since we use the lagged VIX, this aspect of our empirical investigation suggests practical applications.

Third, we estimate a two-state, regime-shifting model that allows for the relation between stock and bond returns to vary between regimes. We find that the first regime exhibits a strong positive co-movement between stock and bond returns and relatively high average stock returns. This regime can be characterized as a low or decreasing VIX regime with low time-series variability in the VIX. Approximately 55% of the daily observations are precisely estimated to be in this regime.² In the second regime, stock and bond returns exhibit a reliable negative co-movement, average bond returns are relatively high, and VIX tends to be high or increasing. Approximately 20% of the daily observations are precisely estimated to be in this regime.

Finally, we investigate the contemporaneous relation between *changes in VIX* and bond returns, while controlling for the stock return. In contrast to the very large negative relation between stock returns and changes in VIX, we find a reliable positive relation between bond returns and the contemporaneous change in VIX.

²By precisely estimated, we mean the smoothed probability of being in a particular regime is 80% or greater. About 25% of the observations do not clearly fall in either regime.

Taken together, our findings suggest that stock market uncertainty has an important role in understanding stock and bond return co-movements. Our findings also suggest that the implied volatility from equity index options may prove useful for financial applications that need to understand and predict stock and bond market co-movements.

Further, our findings imply that diversification benefits increase for portfolios of stocks and bonds during periods of high stock market uncertainty. The timeliness of this increased diversification benefit is in contrast to cross-equity market diversification, where much of the literature has argued that cross-market equity return relations are more positive during times of market stress.

The remainder of this study is organized as follow. Section 2 provides additional background and discusses related literature. Section 3 describes our data. Section 4 examines how stock and bond prices move as a direct function of the lagged VIX. Section 5 studies the relation between stock and bond returns with a regime-switching model. Finally, Section 6 examines how bond returns co-vary with contemporaneous changes in VIX. Section 7 concludes.

2 Background and related literature

Campbell and Ammer (1993) consider traditional fundamentals and discuss several offsetting effects behind the correlation between stock and bond returns. First, variation in real interest rates may induce a positive correlation between stock and bond returns since the prices of both assets are effected by changes in the discount rate. Second, variation in expected inflation may induce a negative correlation between stock and bond returns since increases in inflation are bad news for bonds and ambiguous news for the stock market. Third, common movements in future expected returns may induce a positive correlation between stock and bond returns. The net effect in their monthly return sample over 1952 to 1987 is a small positive correlation between stock and bond returns ($\rho = 0.20$).

In Fleming, Kirby, and Ostdiek (1998), they consider two distinct effects when evaluating volatility linkages between the stock and bond markets. First, common information may affect expectations and the valuation of both the stock and bond markets. Second, there may be a cross-market hedging effect, where cross-market hedging refers to changes in the demand for bonds,

based on information events that alters expectations about stock returns. This change in demand for bonds may occur even if there is no changes in expectations about interest rates. They estimate a model that takes both these effects into account and find that information linkages in the stock and bond markets may be greater than previously thought. Relatedly, Busse (1999) and Fleming, Kirby, and Ostdiek (2001) provide evidence that volatility timing has economic value.

The idea that uncertainty about the economic state may impact return dynamics is suggested in Veronesi (1999) and (2001). In Veronesi (1999), the economy is modelled as a two-state economy where the drift in future dividends shifts between unobservable states. During times of higher uncertainty about the state, new information may receive relatively higher weighting which may induce time-varying volatility and volatility clustering. In Veronesi (2001), the idea of “aversion to state-uncertainty” is introduced. In this paper, the economy may exhibit structural breaks, which generate time-variation in investors’ belief about the dispersion in the distribution of the underlying drift rate of dividends. Regarding bonds and stock volatility, he states, “Intuitively, aversion to state-uncertainty generates a high equity premium and a high return volatility because it increases the sensitivity of the marginal utility of consumption to news. In addition, it also lowers the interest rate because it increases the demand for bonds from investors who are concerned about the long-run mean of their consumption.”

In our view, the idea of pricing influences associated with economic uncertainty and flight-to-quality may be interrelated with the idea of cross-market hedging, as proposed in Fleming, Kirby, and Ostdiek (1998). We extend this prior work by using the VIX as a directly observable and dynamic measure of stock market uncertainty. We then explore whether the co-movement between stock and bond returns is related to this notion of stock market uncertainty.

3 Data

3.1 Basic data description

We examine daily data over the 1988 to 2000 period in our analysis. We choose the daily horizon for several reasons. First, sizable changes in stock market uncertainty may occur intraday. For

example, in our sample of 3251 trading days, the CBOE’s Volatility Index (VIX) changes by 15% or more for 84 different days and by 10% or more for 279 different days.³ Second, it seems plausible that the attractiveness of bonds, relative to stocks, may also experience significant changes within a single day. Finally, the use of daily data provides many more observations for our econometric estimation and is suggested by prior studies such as Busse (1999) and Fleming, Kirby, and Ostdiek (1999 and 2001).

We focus on the 1988 to 2000 sample period for several reasons. First, Christensen and Prabhala (1998) find that implied volatility is a significantly better predictor of future volatility following the October 1987 crash. Second, this 13-year period provides a substantial sample of 3251 days and includes a recession and periods of international crisis (for example, the Persian Gulf War of 1991 and the Asian and Russian crisis in 1997-98). Third, since the CBOE’s VIX is first reported in 1986, we only lose two years by focusing on the 1988 to 2000 period and we avoid econometric concerns that our empirical results will be dominated by the October 1987 stock market crash.⁴

For our measure of stock market implied volatility, we use the CBOE’s VIX, calculated from the implied volatility of S&P 100 index options. This index, described by Fleming, Ostdiek, and Whaley (1995), represents the implied volatility of an at-the-money option on the S&P 100 index with 22 trading days to expiration. The VIX is constructed by taking a weighted average of the implied volatilities of eight options, calls and puts at the two strike prices closest to the money and the nearest two expirations (excluding options within one week of expiration). Each of the eight component implied volatilities is calculated using a binomial tree that accounts for early exercise and dividends.⁵

For daily bond returns, we analyze both 10-year U.S. Treasury notes and 30-year U.S. Treasury bonds. We calculate implied returns from the constant maturity yield from the Federal Reserve.

³By a change, we mean $(VIX_t - VIX_{t-1})/VIX_{t-1}$, where VIX_t is the implied volatility level at the end-of-the-day.

⁴In addition to the extreme stock return of -17% on October 19, 1987, the implied volatility of equity index options exceeded 100% (annualized standard deviation units) for a few days around the crash. We do include the 1986-87 period during analysis of alternate periods that are reported in the Appendix.

⁵In calculating the VIX, each option price is calculated using the midpoint of the most recent bid/ask quote to avoid bid/ask bounce issues. The VIX construction uses four calls and four puts to minimize mis-measurement concerns and any put/call option clientele effects.

Hereafter, we do not distinguish between notes and bonds in our terminology and refer to both the 10-year note and the 30-year bond as “bonds”. We choose longer-term securities over shorter-term securities because monetary policy operations are more likely to have a confounding influence on shorter-term securities.

Fleming (1997) characterizes the market for U.S. Treasury securities as “one of the world’s largest and most liquid financial markets.” Using 1994 data, he estimates that the average daily trading volume in the secondary market was \$125 billion. Fleming also compares the trading activity by maturity for the most recently issued securities. He estimates that 17% of the total trading is in the 10-year securities and 3% of the total trading is in the 30-year securities.

For robustness, we also evaluate a return series from the Treasury bond futures contract that is traded on the Chicago Board of Trade. To construct these returns, we use the continuous futures price series from Datastream International from 1988 through 2000. This series switches to a new contract as the nearby contract enters the delivery month.

For the aggregate stock market return, we use the value-weighted index of NYSE/AMEX/NASDAQ from the Center for Research in Security Prices (CRSP). When merging the stock and bond returns, we find that there are a number of days where there is not an available yield for the debt securities. These appear to be largely on Federal holidays where the stock market was still open. After deleting these days with missing values, we have 3251 observations for each data series. All returns are in daily percentage terms.

Panel A of Table 1 reports basic univariate statistics for the data series over the 1988 to 2000 period. S, B10, and B30 refer to the stock, 10-year Treasury bond, and 30-year Treasury bond return series, respectively. DVIX stands for the daily change in the implied variance from the CBOE’s VIX.

Table 1, Panel B, reports the simple correlation between the variables. We note that the unconditional correlation between stock and T-bond returns is modest at 0.22 (10-year bonds) and 0.25 (30-year bonds). This correlation is consistent with prior literature that has puzzled over the modest correlation between stock and bond returns. Second, we find that both VIX and DVIX are negatively correlated with stock and T-bond returns. Of particular interest, the correlation

between DVIX and stock returns is very substantial at -0.712. In contrast, the correlation between DVIX and the 10-year T-bond returns is near zero at -0.056.

3.2 Description of bond and stock return volatility

In our empirical investigation, we examine whether time-variation in expected stock market volatility has implications for the relation between bond and stock returns. Here, for perspective, we first provide a brief comparison of the daily volatility in stock and 10-year T-bond returns. For our 1988 to 2000 sample, the unconditional daily variance of the T-bond returns is only about one-fourth as large as the unconditional daily variance of stock returns.

Next, we estimate a time-series of conditional volatilities for the stock and bond return series for comparison. For this discussion, conditional volatility refers to the conditional standard deviation, estimated by a GARCH(1,1) model that includes the lagged VIX as an explanatory term in the variance equation. We find that the time-variation in stock conditional volatility is much larger than the time-variation in bond conditional volatility. For our sample, the time-series standard deviation of the bond conditional volatility is only about one-sixth as large as the time-series standard deviation of the stock conditional volatility. Finally, we note that the correlation between stock and bond conditional volatility is a modest 0.176.

3.3 Predictability of bond and stock returns in a VAR framework

In the next section, we are interested in how the innovation in T-bond prices moves with the innovation in stock prices. If the stock and bond returns are predictable, then we should first control for this predictability before examining the relation in the return innovation. One possibility is to first estimate a vector autoregression (VAR) in the bond returns and stock returns. Then, the residuals from the VAR could be examined in order to evaluate how the return innovations co-vary.

However, daily stock and bond returns exhibit very little predictability. We estimate a 5-lag VAR system for the stock and bond return series over our 1988 to 2000 sample. For this model, the adjusted R-squared's are 0.64%, 1.04%, and 0.48%, respectively, for the stock returns, the 10-year bond returns, and the 30-year bond returns.

In this VAR estimation, the most reliable lagged explanatory variable for the stock and bond returns is their own first-lag. Thus, in the subsequent empirical work, we report on simple models that use the raw variables (rather than VAR residuals) and that include the first-lag of the dependent variable as an additional explanatory variable to control for the modest autoregressive predictability.

It is important to note that our regression models are not meant to imply formal economic causality between the dependent and independent variables. Rather, the regression models are meant to examine the co-movements between stock and bond returns and the role of stock market uncertainty. Since we are primarily interested in whether expected stock volatility has cross-market pricing influences, we use the bond return as the dependent variable in our regression models. We note that an analysis using residuals from the VAR estimation yields essentially identical results.

4 Variation in the relation between stock and bond returns as a function of lagged VIX

4.1 Variation in the simple correlation between stock and bond returns, with observations sorted by the lagged VIX

We first investigate how the correlation between daily stock returns and Treasury bond returns varies with the lagged VIX by sorting our return sample on VIX_{t-1} . We then calculate the stock-bond correlation for each VIX_{t-1} quintile of observations.

The results are reported in Table 2, Panel A. We find that the largest VIX_{t-1} quintile has the smallest correlation between stock and bond returns at values of $\rho_{S,B10} = 0.015$ and $\rho_{S,B30} = 0.078$. In contrast, the smallest VIX_{t-1} quintile has the largest correlation at values $\rho_{S,B10} = 0.443$ and $\rho_{S,B30} = 0.472$.

We further subdivide the largest VIX_{t-1} quintile into the largest and second largest VIX_{t-1} deciles. For the 10-year bonds, $\rho_{S,B10} = -0.056$ ($\rho_{S,B10} = 0.101$) for the largest (second largest) VIX_{t-1} decile. For the 30-year bonds, $\rho_{S,B30} = -0.005$ ($\rho_{S,B30} = 0.160$) for the largest (second largest) VIX_{t-1} decile. Thus, the measured correlation between stock and bond returns for the

highest VIX_{t-1} decile is not only weaker than the lower VIX subsets, but it even becomes negative.

These results also indicate a striking absence of a positive risk-return relation for the stock returns. Note that the largest VIX_{t-1} quintile of stock return observations has, by far, the largest realized volatility (as one would expect). However, the average daily stock return for this quintile is 0.049% per day, which is lower than the unconditional daily average of 0.061% per day.

Finally, these results indicate that the bond return volatility varies little with the VIX_{t-1} . For the largest VIX_{t-1} quintile, the standard deviation of the stock returns is 1.41% per day, versus the unconditional standard deviation of 0.89% per day. In contrast, for this same quintile, the standard deviation of the 10-year (30-year) bond returns is 0.45% per day (0.71% per day) versus the unconditional standard deviation of 0.41% per day (0.63% per day).

4.2 Hypothetical correlations under two simple benchmarks

Next, in Panel B of Table 2, we report the hypothetical correlation between the stock and bond returns for each VIX_{t-1} quintile under two alternate simple benchmarks. We are not arguing that either benchmark is necessarily a good assumption, but we believe this exercise provides an interesting comparison.

Benchmark One. The motivation for this benchmark follows from the intuition in Forbes and Rigobon (2000). They consider the case where return-series 'Y' is economically related to return-series 'X', such that the relation can be represented by a constant coefficient found by regressing 'X' on 'Y'. They note that if series 'X' exhibits heteroskedasticity and the remaining random idiosyncratic component of series 'Y' is homoskedastic (or at least less heteroskedastic), then the measured correlation between 'Y' and 'X' should be higher during periods with high series 'X' volatility. They demonstrate that this feature may lead researchers to erroneously conclude that markets are more linked during times of high market volatility.

To illustrate, following from Forbes and Rigobon (2001), assume that x and y are stochastic variables which represent stock and bond market returns, respectively. Further, assume that the returns have an economic relation that can be represented by the following equation.

$$y_t = \alpha + \beta x_t + \varepsilon_t \tag{1}$$

where $E[\varepsilon_t] = 0$, $E[x_t\varepsilon_t] = 0$, the variance of ε is some finite constant, and α and β are estimated coefficients (assumed to be constant) that represent economic parameters. The stock return, x_t , exhibits heteroskedasticity over time, which is denoted below by the t subscript for the variance of x . Under this set of assumptions, the correlation between y and x can be given by:

$$\rho_{x,y} = \frac{\beta\sigma_{x,t}}{\sqrt{\beta^2\sigma_{x,t}^2 + \sigma_\varepsilon^2}} = \frac{\beta\sigma_{x,t}}{\sigma_{y,t}} \quad (2)$$

As a result, the measured correlation between x and y should increase during times when the variance of x is high, even if the true relationship (the β) between x and y is constant. This is because the numerator in (2) increases proportionately more than the denominator as the variance of x increases.

We use equation (2) to calculate the hypothetical correlations in Panel B of Table 2, denoted "Benchmark One". We assume a constant relation between the bond and stock returns and use the β from an OLS regression of (1), estimated on the entire sample. For the stock and bond return volatility in (2), we use the respective sample volatility for each VIX_{t-1} quintile as reported in Panel A of Table 2.

Note the contrast between the actual stock-bond correlations in Panel A and these hypothetical correlations in Panel B. For the largest VIX_{t-1} quintile, the hypothetical correlation is 0.315 versus an actual sample correlation of 0.015 for the 10-year bond returns. For the smallest VIX_{t-1} quintile, the hypothetical correlation is 0.122 versus an actual sample correlation of 0.443 for the 10-year bond returns. Thus, this first simple benchmark is clearly far off the mark and indicates that the economic relation between stock and bond returns cannot be represented by the simple relation in (1).

Benchmark Two. Of course, an alternate benchmark is that the expected stock return can be expressed as a fixed proportion of the bond return. Under Benchmark Two, the high heteroskedasticity in stock returns is reflected in time-varying volatility in the stock's random idiosyncratic component. Here, the setup is:

$$x_t = \alpha + \beta y_t + \varepsilon_t \quad (3)$$

where $E[\varepsilon_t] = 0$, $E[y_t\varepsilon_t] = 0$, the variance of ε is time-varying, and α and β are estimated coefficients

(assumed to be constant) that represent economic parameters. The bond return, y_t , is assumed to be homoskedastic (approximately). Under this set of assumptions, the correlation between y and x can be given by:

$$\rho_{x,y} = \frac{\beta\sigma_y}{\sqrt{\beta^2\sigma_y^2 + \sigma_{\varepsilon,t}^2}} = \frac{\beta\sigma_y}{\sigma_{x,t}} \quad (4)$$

Under this alternate benchmark, the correlation between stock and bond returns should decrease with higher stock volatility. This is because the higher stock volatility is attributed to the higher volatility of the idiosyncratic component, ε , which increases only the denominator in (4).

We use equation (4) to calculate the hypothetical correlations in Panel B of Table 2, denoted "Benchmark Two". The β is from an OLS regression of (3), estimated on the entire sample. For the stock and bond return volatility in (4), we use the respective sample volatility for each VIX_{t-1} quintile. Here, the hypothetical correlations are $\rho_{S,B10} = 0.150$ for the largest VIX_{t-1} quintile and $\rho_{S,B10} = 0.388$ for the smallest VIX_{t-1} quintile. This difference in hypothetical correlations between the smallest and largest VIX_{t-1} quintile is 0.238, versus the actual difference of 0.428 reported in Panel A of Table 2. Thus, while this simple benchmark seems to fit the actual data better, the variation in correlations is still less than observed in the actual data. Further, neither of the two simple benchmarks in this subsection is capable of explaining a conditional negative correlation between stocks and bonds.

4.3 Variation in the relation between stock and bond returns as a function of lagged VIX in a simple parametric model

Next, we estimate the following GARCH model that allows the relation between stock and bond returns to vary directly (and continuously) with the log of the lagged VIX level. We estimate two variations of the model, one using the VIX from period $t - 2$ and the other using the VIX from period $t - 22$ (one month old).

$$B_t = a_0 + a_1 B_{t-1} + (a_2 + a_3 \ln(VIX_{t-k})) S_t + a_4 \ln(VIX_{t-k}) + \varepsilon_t, \quad (5)$$

$$h_t = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \gamma_2 h_{t-1} + \gamma_3 VIX_{t-2}, \quad (6)$$

where B_t denotes the T-bond return in period t , S_t is the stock return in period t , ϵ_t is the residual, $\ln(VIX_{t-k})$ is the natural log of the VIX level at time $t-k$, k equals either 2 or 22 days, and the a_i 's and γ_i 's are estimated coefficients. We use the log transformation of VIX to reduce the skewness of the implied volatility series. The above system is estimated simultaneously by maximum likelihood using the conditional normal density.

We use the $t-2$ VIX (rather than the $t-1$) to provide clear temporal separation between the lagged VIX and the period t returns. For example, this choice avoids any concern that price measurement issues may introduce some related error in the VIX value and the period t return (since both rely on stock prices from the end of day $t-1$). By using the one-month old VIX, we can comment on the horizon for forecasting variations in the stock-bond return relation.

We include the lagged VIX level by itself (the a_4 term) as an additional explanatory variable to control for any direct relation between bond returns and the lagged VIX. Since the VIX term is lagged, a positive a_4 might indicate a gradual (rather than instantaneous) re-valuation of bonds, relative to stocks, during periods of high stock market uncertainty. Or, if VIX represents overall economic uncertainty, a positive a_4 might indicate a positive risk-return tradeoff for bond returns. However, since stock returns do not reflect a positive risk-return tradeoff with lagged VIX (see Section 4.1), it seems unlikely that bond returns would.

The results from this model are reported in Table 3. Panel A reports the results for the 10-year Treasury bond returns and Panel B reports the results for the 30-year Treasury bond returns.

We find that the relation between the stock and bond returns varies negatively and very reliably with the lagged VIX. The variation in the stock-bond return relation appears substantial. The last three rows in each panel report the total implied coefficient on the stock return at the median, 95th percentile, and 5th percentile of the lagged VIX value, respectively. For the 10-year T-bonds, the total implied coefficient on the stock return is substantial at values of around 0.352 for the 5th percentile of the lagged VIX. In contrast, at the 95th percentile of the lagged VIX, the total implied coefficient on the stock return is near zero at 0.012. The results for the VIX_{t-22} are qualitatively similar.

For the 30-year bonds, we find that the relation between the stock and bond returns also varies

negatively and very reliably with the lagged VIX. The variation in the stock-bond relation is even larger for the 30-year T-bonds. At the 5th percentile of the lagged VIX, the total implied coefficient on the stock return is substantial at 0.567. In contrast, at the 95th percentile of the lagged VIX, the total implied coefficient on the stock return is close to zero at around 0.044. Thus, the results for the 30-year T-bond series reinforce our results for the 10-year T-bond series.

We note that the estimated a_4 coefficients for this model are all positive but statistically insignificant. In the Appendix, we report on estimating a similar model over four alternate periods. The estimated a_4 's are positive in all four periods and positive and statistically significant in the July 1993 through December 2000 subperiod. Thus, while the statistical reliability is suspect, at least the sign of these coefficients is consistent with the idea that bond prices may increase, relative to stocks, during times of high stock market uncertainty.

The results in Tables 2 and 3 also imply something about the diversification value from holding both stocks and bonds. Our findings suggest that, during times of high uncertainty in the stock market, the relation between stock and bond returns appears much weaker. Further, the lagged VIX is not reliably related to time-varying bond volatility (note that the estimated γ_3 coefficients are statistically insignificant in the full model of Table 3). These characteristics suggest an increased diversification benefit from holding bonds with stocks, exactly during the times when diversification is most needed. In contrast, several papers have suggested that cross-market equity returns become more correlated during times of market shocks or economic uncertainty.⁶

4.4 Robustness: Returns from Treasury bond futures prices

We also examine the T-bond futures-return series for robustness. The futures series behaves similarly to the 30-year bond-return series from the Federal Reserve data. The correlation between the daily futures-return series and the 30-year bond-return series is 0.955 over the 1988 to 2000 period. Further, the volatility is comparable with the futures-return series having a daily standard deviation of 0.568% per day versus 0.633% per day for the 30-year bond series in Table 1. The

⁶For example, King and Wadhwani (1990) and Lee and Kim (1993) suggest that the cross-market equity correlation increased following the U.S. market crash of 1987. However, note that Forbes and Rigobon(2001) cast some doubt on these findings.

correlation between the futures-return series and the stock market return is 0.270, similar to the 0.250 correlation between the stock return and our 30-year bond-return series. Finally, we find that the results from estimating the system in (5) and (6) with the bond-futures returns are very similar to the results for the 30-year bond returns.

5 Variation in bond and stock return co-movements:

A regime-shifting approach

5.1 Background and motivation

The results from the previous section support the notion that stock market uncertainty (measured by the lagged implied volatility from equity index options) is associated with substantial variation in the dynamic relation between stock and bond returns. In this section, we extend our analysis in a second dimension. We examine whether the regime-switching framework of Hamilton (1989) might be useful in characterizing the dynamic relation between bond returns and stock returns.⁷

Accordingly, in this section, we investigate whether regime-switching behavior is exhibited in: (1) the relation between daily T-bond and stock returns, and (2) the average T-bond returns, relative to stock returns. If so, we are also interested in whether the regimes are associated with variation in VIX. Our model allows the transition probabilities to be a function of the lagged VIX.

We evaluate a regime-shifting approach for the following reasons. First, as previously discussed, prior literature suggests that this approach may prove useful. Second, in our view, the time-series behavior of the VIX suggests that a regime-switching approach may be worthwhile. See the upper time-series in Figure 1. Casual inspection of the VIX series suggests several regions of distinctly different behavior. First, from 1/88 to about 9/89, the VIX decreases and then levels off following the October 1987 crash. From around 10/89 to about 10/92, the VIX exhibits substantial variation with several periods of high implied volatility. Then, from around 11/92 to about 2/96,

⁷See Hamilton (1994), Chapter 22, for a review of regime-switching processes and related literature. Boudoukh, Richardson, Smith, and Whitelaw (1999) argue that regime shifts are useful in understanding bond returns. Hamilton and Susmel (1994) and Whitelaw (2000) study regime shifts in stock volatility. See also Hamilton and Lin (1996) and Veronesi (1999) and (2001).

the VIX is lower and exhibits relatively low variability. Next, from 3/96 through 9/97 the VIX increases modestly. Finally, from 10/97 through 2000, the VIX tends to be high with very high variability. While any such description has some subjectivity, the regions of different VIX behavior suggest possible regime-shifting behavior related to variation in stock market uncertainty. Third, a regime-shifting approach is evaluated because it does not constrain the stock-bond return relation to be related to the lagged VIX in a continuous parametric function, such as in our model (5). Thus, a regime-shifting approach seems closer to the intuition of an occasional decoupling in stock and bond returns during periods of economic uncertainty or market distress, as suggested by the aforementioned Wall Street Journal articles.

5.2 The regime-switching model and results

We estimate the following two-state regime-switching model, specified as follows:

$$B_t = a_0^s + a_1 B_{t-1} + a_2^s S_t + \epsilon_t, \quad (7)$$

where ϵ_t is a Gaussian innovation, s can be regarded as an unobserved state variable that follows a two-state, first-order Markov process, and the other terms are as defined for equation (5). The transition probability matrix is expressed as follows:

$$\mathbf{X} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix} \quad (8)$$

where $p = Pr(s_t = 0 | s_{t-1} = 0; I_{t-1})$, and $q = Pr(s_t = 1 | s_{t-1} = 1; I_{t-1})$. I_{t-1} is the information set.

In stead of letting p and q be constants as is the case of Hamilton (1989), we choose to let these transition probabillites be time-varying. Following Diebold et al. (1994), the time-varying transition probabilities are specified as follows:

$$p(s_t = j | s_{t-1} = j; I_{t-1}) = \frac{e^{c_j + d_j \ln(VIX_{t-1})}}{1 + e^{c_j + d_j \ln(VIX_{t-1})}}, j = 0, 1. \quad (9)$$

Thus, the transition probabilities are allowed to change with the lagged VIX levels. Hence this model specification is flexible and encompasses a constant transition probabilities model.

We elect not to model heteroskedasticity in the bond returns for parsimony and the following additional reasons. First, time-variation in bond return volatility is much smaller than time-variation in stock return volatility. Further, the correlation between time-varying stock volatility and time-varying bond volatility is modest and the lagged VIX is not reliably related to time-varying bond volatility. See our prior discussion in Sections 3.2, 4.1, and 4.3, and results in Tables 2 and 3.

Our results for this regime-shifting estimation are reported in Table 4. Column one in Table 4 reports results for the 10-year Treasury bonds over the 1988 to 2000 period. In the first regime (denoted regime-zero in the table), we find that the a_2^0 coefficient on stock returns is large and statistically significant at a value of 0.343. The estimated intercept, a_0^0 , is negative but insignificant. In contrast, in the second regime (denoted regime-one in the table), we find that the a_2^1 coefficient on stock returns is negative and statistically significant at a value of -0.061 (a difference of 0.40, as compared to regime-zero). For regime-one, the estimated intercept, a_0^1 , is positive and highly statistically significant.

The results for the 30-year T-bonds are qualitatively similar but even stronger in magnitude. For the 30-year T-bonds, the difference between a_2^0 for regime-zero and a_2^1 for regime-one is 0.62. The estimated intercept increases from -0.0303 in regime-zero to a highly reliable 0.0924 in regime-one.

For the transition probabilities, we note the following. First, we note that the estimated c_0 's are sizeable and significantly positive. More interestingly, the estimates of d_0 are significantly negative, which indicates that an increase in VIX_{t-1} will lower the probability of staying in regime zero. Hence, regime zero is associated with periods of relatively lower stock market volatilities.

This concept is illustrated by the expected durations reported in the last four rows of Table 4. At a VIX_{t-1} of 15%, the expected duration of staying in regime-zero is 53.1 days for the 10-year bond returns and 67.3 days for the 30-year bonds. In contrast, at a VIX_{t-1} of 30%, the expected duration of staying in regime-zero is only 15.6 days for the 10-year bond returns and 10.0 days for the 30-year bonds.⁸

For regime-one, we find that the estimated c_1 's are much smaller (and statistically insignificant),

⁸The expected duration of regime i is calculated as follows: $E(D) = \frac{1}{1-p_{ii}}$, $p_{ii} \equiv Pr(s_t = i | s_{t-1} = i)$.

as compared to the c_0 's. Next, in contrast to the negative d_0 's, the estimated d_1 's are positive (but statistically insignificant). These point estimates for c_1 and d_1 imply the following expected durations for regime-one. At a VIX_{t-1} of 15%, the expected duration of staying in regime-one is only 12.7 days for the 10-year bond returns and 13.9 days for the 30-year bonds. In contrast, at a VIX_{t-1} of 30%, the expected duration of staying in regime-zero is 34.4 days for the 10-year bond returns and 19.1 days for the 30-year bonds.

Next, we further describe each regime. For the purposes of this description, we categorize an observation as regime-zero if there is an 80% chance or greater that the observation is in regime-zero. Likewise, we categorize an observation as regime-one if there is an 80% chance or greater that the observation falls in regime-one.

Under this classification convention, 53.6% (54.7%) of the daily observations are categorized as regime-zero for the 10-year bond model (30-year bond model). On the other hand, 20.6% (18.6%) of the daily observations are categorized as regime-one for the 10-year bond model (30-year bond model). Thus, about 25% of the observations are not clearly classified into either regime.

Next, using this classification convention, we calculate basic descriptive statistics for the return observations in each regime. First, as suggested by the negative d_0 coefficient, we note that regime-zero has a lower stock volatility with a stock return standard deviation of only about 0.7%/day. In contrast, for the regime-one observations, the standard deviation of stock returns is about 1.4%/day. The volatility of the bond returns varies little across the two regimes. For both the 10-year and 30-year bond returns, the bond return volatility in each regime is within 10% of the series' unconditional volatility.

Next, we contrast Sharpe ratios for the stock and bond returns across the two regimes. For regime-zero in the 10-year (30-year) bond model, the daily Sharpe ratio is 0.087 (0.082) for the stock returns and -0.032 (-0.021) for the bond returns. In contrast, for regime-one in the 10-year (30-year) bond model, this ratio is 0.001 (0.009) for the stock returns and 0.104 (0.174) for the bond returns. Thus, in regime-zero, stocks are less volatile, stocks substantially outperform bonds (in a Sharpe ratio sense), and stock and bond returns exhibit a substantial positive co-movement relation. In contrast, in regime-one, stocks are highly volatile, bonds substantially outperform stocks, and stock

and bond returns exhibit a reliable negative co-movement relation. These characteristics suggests the intuition of referring to regime-zero as a 'low stock market uncertainty' regime, and referring to regime-one as a 'high stock market uncertainty' regime.

For comparison, we have also estimated a version of the model that assumes constant transition probabilities (restricts the d_j 's=0). For this variation of the model, the estimated a_0^s 's and a_2^s 's remain very similar to those shown in Table 4. The regime movements display similar characteristics but the expected durations are somewhat longer for each regime. For example, for the 30-year bond returns in the constant transition probability model, the expected duration is 65.8 days for regime-zero and 28.9 days for regime-one. We perform a likelihood ratio test that compares the constant transition probability model to our time-varying transition probability model in (7) and (8). This test rejects the constant transition probability model with a p-value of less than 0.001 for the 10-year bond and a p-value of 0.053 for the 30-year bond.

5.3 Regime-switching and the VIX series

The results from estimating the regime-shifting model of (7) and (8) indicate that the VIX is a statistically significant explanatory variable when modelling the transition probabilities, at least for regime-zero. We next present the relation between VIX movements and the regimes graphically. In Figure 1, the upper series is the VIX and the lower series is the smoothed probability of being in regime-one for the 10-year T-bond returns. In the 1988 to 2000 period, regime-zero is the predominant regime from about 3/88 to 8/89 and from 3/93 to 9/97. These time periods tend to be either decreasing or low VIX periods with relatively less day-to-day variation in VIX.

In contrast, regime-one spans much of the 1/88 to 2/88, 9/89 to 2/93 and 10/97 to 12/00 periods, with occasional regime-zero interim periods. Overall, regime-one seems to be associated with a high VIX and/or substantial day-to-day variability in VIX.

Figure 2 presents the same comparison for the regime behavior for the 30-year Treasury bond return series. The results are comparable to those for the 10-year bond, as described in the preceding paragraph.⁹

⁹In our previous discussion and in Figures 1 and 2, we focus on the smoothed probabilities for the regime move-

Eye-ball statistics from Figures 1 and 2 suggest that there exists a relation between the regime probabilities and the VIX series. To further illustrate this relation, Figure 3 shows the scatterplot of the regime-one probability against the VIX level. This figure indicates that the specific functional form of this relation is hard to capture parametrically. Hence, we resort to nonparametric estimation techniques to fit the relation for the illustration in Figure 3. We use a method called “local polynomial fitting”.¹⁰ The solid line is a fitted curve estimated nonparametrically using local polynomial fitting with degree 1. Note that the curve is strongly upward-sloping, clearly illustrating the association between regime-one and higher stock market uncertainty.

5.4 Comparison of inflation and short-term interest rates across the two regimes

Campbell and Ammer (1993) consider fundamental factors that may jointly determine movements in stock and bond returns. They note that movements in real interest rates should induce a positive correlation between stock and bond returns. On the other hand, movements in inflation should induce a negative correlation between stock and bond returns. Thus, time-variation in the movements in real rates and inflation might conceivably induce regime-shifts in the co-movement between stock and bond returns.

In this subsection, we describe monthly movements in inflation and real short-term interest rates across the regimes. For this exercise, we smooth the regime movements and categorize months from 1/88 to 2/88, 10/89 to 2/93, and 10/97 to 12/00, as predominantly regime-one months. All of the other months are categorized as predominantly regime-zero. For inflation, we evaluate monthly changes in the seasonally-adjusted Consumer Price Index. For the monthly short-term real interest rate, we use the difference between the yield on 3-month T-bills and the month’s inflation.

First, we report the inflation comparison. For the entire thirteen year sample, the average

ments. The results for the filtered probabilities are similar but somewhat more noisy.

¹⁰The basic idea of local polynomial fitting is very simple: in a neighborhood of an observation, say x_0 , a polynomial of degree p is fitted to the data. Compared with other popular nonparametric smoothing techniques, especially kernel regression, local polynomial fitting is known to have better small sample properties, see e.g., Fan and Gijbels (1996). In fact, the classical Nadaraya-Watson kernel regression corresponds to the special case of a local polynomial regression with degree zero.

inflation was 0.264% per month and the average absolute change in the month-to-month inflation rate was 0.146%. For the predominantly regime-zero months, the average inflation was 0.273% per month and the average absolute change in the month-to-month inflation rate was 0.123%. For the predominantly regime-one months, the average inflation was 0.255% per month and the average absolute change in the month-to-month inflation rate was 0.167%.

Next, we report the short-term interest rate comparison. For the entire thirteen year sample, the average real rate was 0.193% per month and the average absolute month-to-month change in the monthly T-bill yield was 0.0129% per month. For the predominantly regime-zero months, the average real rate was 0.161% per month and the average absolute month-to-month change in the monthly T-bill yield was 0.0134% per month. For the predominantly regime-one months, the average real rate was 0.223% per month and the average absolute month-to-month change in the monthly T-bill yield was 0.0126% per month.

Based on these cross-regime comparisons, it seems unlikely that the regime-shifting behavior is attributable to differences in the behavior of inflation or the short-term discount rate across the two regimes. Indirectly, this finding seems to support the idea that stock market uncertainty influences cross-market return dynamics.

5.5 Discussion of results

Our collective results suggest that implied equity volatility may serve as a state variable for understanding variations in the stock-bond return relation. Our evidence suggest that stock and bond returns tend to move together substantially during periods of lower stock market uncertainty. However, higher stock market uncertainty is associated with little co-movement or even a negative co-movement between stock and bond returns.

For our regime-shifting model, it is perhaps puzzling that the negative relation between stock and bond returns is exhibited over a sustained period during regime-one. This finding seems at odds with a simple flight-to-quality interpretation that assumes that stock and bond prices move in the opposite direction only briefly during the period when stock uncertainty increases. Rather, our findings suggest that there may be sustained periods of negative correlation between stock

and bond returns, possibly because investors may be frequently revising their estimates about the economic state during times of higher uncertainty. This notion seems consistent with the intuition from Veronesi (1999 and 2001) and the second Wall Street Journal article mentioned in our introduction.

In our view, since the expected duration of regime-one is typically only in the 10 to 20 day range, regime-one may be interpreted as a transitional state associated with higher stock market uncertainty. In contrast, the expected duration of regime-zero is much larger at 50 to 60 days (for typical level of VIX). Thus, regime-zero may be interpreted as more normal times, where stock and bond returns may be linked primarily by common fundamentals.

6 Bond returns and changes in stock implied volatility

So far, we have examined whether the contemporaneous relation between stock and bond returns varies with stock market uncertainty, where stock market uncertainty is measured by the lagged *VIX* level. Here, we report on a different aspect of stock and bond return dynamics to supplement our primary findings.

It is known that stock returns are negatively and reliably associated with contemporaneous changes in VIX ($\rho_{S,DVIX} = -0.71$ in our sample), see Fleming, Kirby, and Ostdiek (1995). However, it is not known whether bond returns are related to changes in VIX, after controlling for the period's stock return. To examine this issue, we estimate the following GARCH(1,1) model to examine the relation between bond returns, stock returns, and *contemporaneous changes* in VIX.

$$B_t = \alpha_0 + \alpha_1 B_{t-1} + \alpha_2 S_t + \alpha_3 DVIX_t + \epsilon_t, \quad (10)$$

$$h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_2 h_{t-1}^2. \quad (11)$$

where B_t denotes the T-bond return in period t , S_t is the contemporaneous stock return, $DVIX_t$ is the change in the implied variance from the CBOE's VIX between the end of period $t - 1$ and the end of period t , ϵ_t is the residual, h_t denotes the conditional variance, and the α_i 's and γ_i 's are estimated coefficients. The above system is estimated simultaneously by maximum likelihood with

the conditional normal density. We report t-statistics calculated with quasi-maximum likelihood (QML) standard errors in accordance with Bollerslev and Wooldridge, 1992.

The coefficient of primary interest is α_3 . If positive, it suggests that bond prices tend to increase, relative to stocks, during days when the stock market uncertainty increases. The model is estimated for both the 10-year and 30-year Treasury bonds.

Table 5 reports the results. For both the 10-year and 30-year T-bond returns, the estimated coefficients for DVIX (α_3) are significantly positive. We conclude that T-bond returns are positively and reliably associated with changes in VIX, after controlling for the stock return. This finding seems consistent with the idea of cross-market hedging (or flight-to-quality) during periods when stock market uncertainty increases.

7 Conclusion

We examine how the co-movement between daily stock and Treasury bond returns varies with stock market uncertainty. We use the lagged implied volatility from equity index options (the CBOE’s VIX) to provide an objective, observable, and dynamic measure of stock market uncertainty. We find that stock and bond returns tend to move substantially together during periods of lower stock market uncertainty. However, stock and bond returns tend to exhibit little relation or even a negative relation during periods of high stock market uncertainty.

We find evidence of these return dynamics in the following alternate approaches in examining the data: (1) a simple correlation analysis, (2) a GARCH specification that allows the relation between stock and bonds to vary directly and continuously with the lagged VIX, and (3) a regime-shifting model that allows the relation between stocks and bonds to vary across regimes. The regime-shifting model also indicates that average bond returns are relatively higher in the regime with a negative co-movement between daily stock and bond returns. In contrast, average stock returns are relatively higher in the regime with a high positive co-movement between daily stock and bond returns.

Finally, after controlling for the period’s stock return, we find a reliable positive relation between daily bond returns and the contemporaneous change in VIX. This finding is in contrast to the very

large negative relation between stock returns and contemporaneous changes in VIX.

Our findings have implications for understanding the joint price formation process for stocks and bonds. First, as suggested in Fleming, Kirby, and Ostdiek (1998), our results are consistent with the notion that cross-market hedging may generate time-varying demand for bonds and an associated influence on the joint return dynamics between stocks and bonds.

Next, our results might be interpreted from the framework of intertemporal asset-pricing. If implied volatility is interpreted as a state variable that is informative about the investment opportunity set or about economic uncertainty (in the sense of Veronesi, 1999 and 2001), then this might explain the link between stock market uncertainty and the time-varying co-movement between stock and bond returns. For example, if the relative attractiveness of bonds versus stocks is volatile during periods of stock market uncertainty, then sustained periods of negative correlation between stocks and bond returns may be observed. In our view, such potential explanations do not replace a cross-market hedging perspective, but rather supplements it. Future research on the economics behind our findings might prove interesting.

Third, our findings imply that models that jointly examine stock and bond returns should allow for the stock-bond return relation to vary with expected stock volatility. In contrast, previous bivariate GARCH models for stock and bond returns has generally assumed a constant correlation between return series (see, e.g. Scruggs (1998)).

From a practical perspective, our results may have direct financial applications. Specifically, implied stock volatility may be a useful state variable for financial applications that need to understand and predict stock and bond market co-movements. For example, our findings suggest increased diversification benefits for portfolios of stocks and bonds during periods of high stock market uncertainty. Such a timely diversification benefit is in contrast to cross-equity market diversification, where much of the literature has argued that cross-market equity returns may be more positively linked during times of high stock market uncertainty.

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Table 1: Descriptive statistics

This table reports the descriptive statistics for the data used in this article. S, B10, and B30 refer to the stock, 10-year Treasury bond, and 30-year Treasury bond return series, respectively. DVIX stands for the daily change in the implied variance from the Chicago Board Option Exchange’s Volatility Index (VIX). Std. Dev. denotes standard deviation and ρ_i refers to the i th autocorrelation. All of the returns are in daily percentage form. The sample period is 1988 to 2000. Panel A reports univariate statistics for each series and Panel B reports the correlation matrix.

| Panel A: Basic Univariate Statistics | | | | | |
|--------------------------------------|--------|--------|--------|--------|--------|
| | VIX | DVIX | S | B10 | B30 |
| Mean | 19.845 | -0.001 | 0.061 | 0.028 | 0.035 |
| Median | 18.690 | -0.003 | 0.084 | 0.021 | 0.021 |
| Maximum | 49.360 | 4.983 | 4.828 | 1.926 | 3.082 |
| Minimum | 9.040 | -3.147 | -6.592 | -2.732 | -3.805 |
| Std. Dev. | 6.293 | 0.322 | 0.892 | 0.414 | 0.633 |
| ρ_1 | 0.973 | -0.160 | 0.060 | 0.075 | 0.032 |
| ρ_2 | 0.953 | -0.102 | -0.023 | -0.004 | 0.015 |
| ρ_3 | 0.937 | -0.055 | -0.036 | -0.044 | -0.029 |

| Panel B: Correlations | | | | | |
|-----------------------|--------|--------|-------|-------|-------|
| | VIX | DVIX | S | B10 | B30 |
| VIX | 1.000 | | | | |
| DVIX | 0.104 | 1.000 | | | |
| S | -0.136 | -0.712 | 1.000 | | |
| B10 | -0.025 | -0.056 | 0.219 | 1.000 | |
| B30 | -0.029 | -0.089 | 0.250 | 0.936 | 1.000 |

Table 2: The correlation between stock and bond returns, conditional on VIX_{t-1}

This table reports the correlation between stock and bond returns for different subsets of return observations. The sample is stratified into quintiles by sorting on VIX_{t-1} . S, B10, and B30 refer to the stock, 10-year Treasury bond, and 30-year Treasury bond return series, respectively. ρ , μ , and σ indicate correlation, mean, and standard deviation, respectively. Panel A reports the actual sample statistics for this sort. Panel B reports hypothetical correlations under two alternate simple benchmarks. Benchmark One assumes that the expected bond return, given the stock return, can be expressed as a constant times the stock return. Benchmark Two assumes that the expected stock return, given the bond return, can be expressed as a constant times the bond return. See Section 4.2 for the complete statistical assumptions and calculation method for these hypothetical correlations.

| Panel A: Sample statistics of the VIX_{t-1} quintiles | | | | | |
|---|-------|-------|-------|-------|-------|
| VIX quintile | Low | 2nd | 3rd | 4th | High |
| $\rho_{S,B10}$ | 0.443 | 0.318 | 0.428 | 0.283 | 0.015 |
| $\rho_{S,B30}$ | 0.472 | 0.365 | 0.411 | 0.300 | 0.078 |
| μ_S | 0.061 | 0.033 | 0.084 | 0.079 | 0.049 |
| σ_S | 0.488 | 0.657 | 0.676 | 0.926 | 1.41 |
| μ_{B10} | 0.038 | 0.028 | 0.020 | 0.026 | 0.029 |
| σ_{B10} | 0.405 | 0.411 | 0.408 | 0.391 | 0.453 |
| μ_{B30} | 0.049 | 0.036 | 0.017 | 0.033 | 0.039 |
| σ_{B30} | 0.611 | 0.592 | 0.607 | 0.638 | 0.711 |

| Panel B: Hypothetical correlations under simple benchmarks | | | | | | |
|--|----------------|-------|-------|-------|-------|-------|
| VIX quintile | | Low | 2nd | 3rd | 4th | High |
| Benchmark One: | $\rho_{S,B10}$ | 0.153 | 0.203 | 0.210 | 0.300 | 0.396 |
| Benchmark Two: | $\rho_{S,B10}$ | 0.388 | 0.293 | 0.283 | 0.198 | 0.150 |

Table 3: The relation between bond returns and stock returns as a function of lagged VIX

This table reports how the relation between daily U.S. Treasury bond returns and stock returns varies with the lagged VIX level. We estimate the following GARCH model:

$$B_t = a_0 + a_1 B_{t-1} + (a_2 + a_3 \ln(VIX_{t-k})) S_t + a_4 \ln(VIX_{t-k}) + \epsilon_t$$

$$h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_2 h_{t-1} + \gamma_3 VIX_{t-2}$$

where B_t and S_t are the daily T-bond and stock returns, respectively; VIX_{t-k} is the CBOE's Volatility Index in period $t-k$; ϵ_t is the residual, h_t is the conditional variance, and the a_i 's and γ_i 's are estimated coefficients. T-statistics are in parentheses, calculated with quasi-maximum likelihood standard errors in accordance with Bollerslev and Wooldridge (1992). The sample period is 1988 to 2000. The last three rows report the total implied coefficient on the stock return at the median, 95th percentile, and 5th percentile of the lagged VIX value, respectively.

| Panel A: 10-Year U.S. Treasury Bonds | | | |
|--|------------------|-------------------|-------------------|
| | Base | k = 2 | k = 22 |
| a_0 | 0.019 (2.91) | -0.032 (-0.53) | -0.046 (-0.64) |
| a_1 | 0.074 (6.18) | 0.066 (4.44) | 0.072 (5.39) |
| a_2 | 0.127 (10.6) | 1.20 (12.2) | 1.14 (6.76) |
| a_3 | | -0.347 (-11.6) | -0.330 (-6.03) |
| a_4 | | 0.017 (0.83) | 0.0217 (0.89) |
| γ_0 | 0.0265 (9.59) | 0.0093 (2.11) | 0.011 (2.30) |
| γ_1 | 0.067 (4.45) | 0.034 (3.85) | 0.040 (4.16) |
| γ_2 | 0.735 (154.5) | 0.891 (21.3) | 0.867 (18.4) |
| γ_3 | 0.0034 (2.66) | 0.0012 (1.21) | 0.0017 (1.27) |
| Likel. Function | 1372.2 | 1463.5 | 1443.1 |
| $a_2 + a_3 \ln(VIX)$ (at the median VIX) | | 0.188 | 0.173 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 95th percentile) | | 0.012 | 0.006 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 5th percentile) | | 0.352 | 0.329 |

Table 3: (continued)

| Panel B: 30-Year U.S. Treasury Bonds | | | |
|--|------------------|-------------------|-------------------|
| | Base | k = 2 | k=22 |
| a_0 | 0.022 (1.76) | -0.048 (-0.45) | -0.073 (-0.71) |
| a_1 | 0.011 (0.69) | 0.018 (1.11) | 0.022 (1.46) |
| a_2 | 0.228 (9.75) | 1.88 (9.00) | 1.85 (7.81) |
| a_3 | | -0.534 (-8.17) | -0.531 (-6.98) |
| a_4 | | 0.023 (0.63) | 0.031 (0.90) |
| γ_0 | 0.019 (0.32) | 0.028 (0.68) | 0.047 (1.08) |
| γ_1 | 0.029 (2.14) | 0.027 (3.12) | 0.0312 (2.83) |
| γ_2 | 0.889 (4.88) | 0.860 (4.94) | 0.769 (4.21) |
| γ_3 | 0.0058 (0.47) | 0.0071 (0.61) | 0.0139 (0.94) |
| Likel. Function | 20.29 | 105.55 | 87.47 |
| $a_2 + a_3 \ln(VIX)$ (at the median VIX) | | 0.314 | 0.292 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 95th percentile) | | 0.044 | 0.023 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 5th percentile) | | 0.567 | 0.544 |

Table 4: A regime-shifting model for the relation between stock and bond returns

This table reports the results for the following regime-switching model.

$$B_t = a_0^s + a_1 B_{t-1} + a_2^s S_t + \epsilon_t,$$

where the regime variable s_t has time-varying transition probabilities:

$$p(s_t = j | s_{t-1} = j; I_{t-1}) = \frac{e^{c_j + d_j \ln(VIX_{t-1})}}{1 + e^{c_j + d_j \ln(VIX_{t-1})}}, j = 0, 1.$$

where I_{t-1} is the information set at $t - 1$, the c_j 's and d_j 's are estimated coefficients, and the other terms are as defined in Table 3. The sample period is 1988 to 2000. T-statistics are in parentheses. The final rows give the expected duration of each regime for different VIX_{t-1} levels.

| | 10-yr T-Bonds | | 30-yr T-Bonds | |
|-------------------------------|---------------|-----------|---------------|---------|
| a_0^0 | -0.0156 | (-1.51) | -0.0303 | (-1.92) |
| a_0^1 | 0.0558 | (4.26) | 0.0924 | (4.28) |
| a_1 | 0.0577 | (3.63) | 0.0102 | (0.642) |
| a_2^0 | 0.3430 | (14.7) | 0.5297 | (13.8) |
| a_2^1 | -0.0609 | (-4.99) | -0.0930 | (-4.54) |
| c_0 | 8.9163 | (2.93) | 11.9900 | (3.83) |
| d_0 | -1.8327 | (-1.86) | -2.8789 | (-2.97) |
| c_1 | -1.6240 | (-0.55) | 1.2433 | (0.42) |
| d_1 | 1.5090 | (1.59) | 0.4853 | (0.53) |
| Expected Duration | | | | |
| $(s_t = 0, VIX_{t-1} = 15\%)$ | | 53.1 days | 67.3 days | |
| $(s_t = 0, VIX_{t-1} = 30\%)$ | | 15.6 days | 10.0 days | |
| $(s_t = 1, VIX_{t-1} = 15\%)$ | | 12.7 days | 13.9 days | |
| $(s_t = 1, VIX_{t-1} = 30\%)$ | | 34.4 days | 19.1 days | |

Table 5: Bond returns and contemporaneous changes in stock implied volatility

This table reports how daily U.S. Treasury bond returns are jointly related to the day's stock returns and the daily change in the VIX level.

$$B_t = \alpha_0 + \alpha_1 B_{t-1} + \alpha_2 S_t + \alpha_3 DVIX_t + \epsilon_t$$

$$h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_2 h_{t-1} + \gamma_3 VIX_{t-1}$$

where $DVIX_t$ is the change in the implied variance from the CBOE's VIX between the end of day $t - 1$ and the end of day t , the α_i 's and γ_i 's are estimated coefficients, and the other terms are as defined in Table 3. The sample period is 1988 to 2000. T-statistics are in parentheses, calculated with quasi-maximum likelihood standard errors in accordance with Bollerslev and Wooldridge (1992).

| | 10-yr T-Bonds | 30-yr T-Bonds |
|------------|------------------|------------------|
| α_0 | 0.018 (3.04) | 0.0204 (1.94) |
| α_1 | 0.060 (4.28) | 0.0136 (1.03) |
| α_2 | 0.183 (13.3) | 0.305 (13.0) |
| α_3 | 0.250 (4.60) | 0.354 (3.88) |
| γ_0 | 0.0056 (2.66) | 0.0094 (2.21) |
| γ_1 | 0.036 (4.55) | 0.028 (4.60) |
| γ_2 | 0.921 (43.4) | 0.934 (42.80) |
| γ_3 | 0.0007 (1.20) | 0.0026 (1.50) |

Figure 1

This figure displays the CBOE's Volatility Index (upper series) and the smooth probability of being in regime-one (lower series) from the regime-shifting model in Table 4 for the 10-year Treasury bond returns. The sample period is 1988 to 2000.

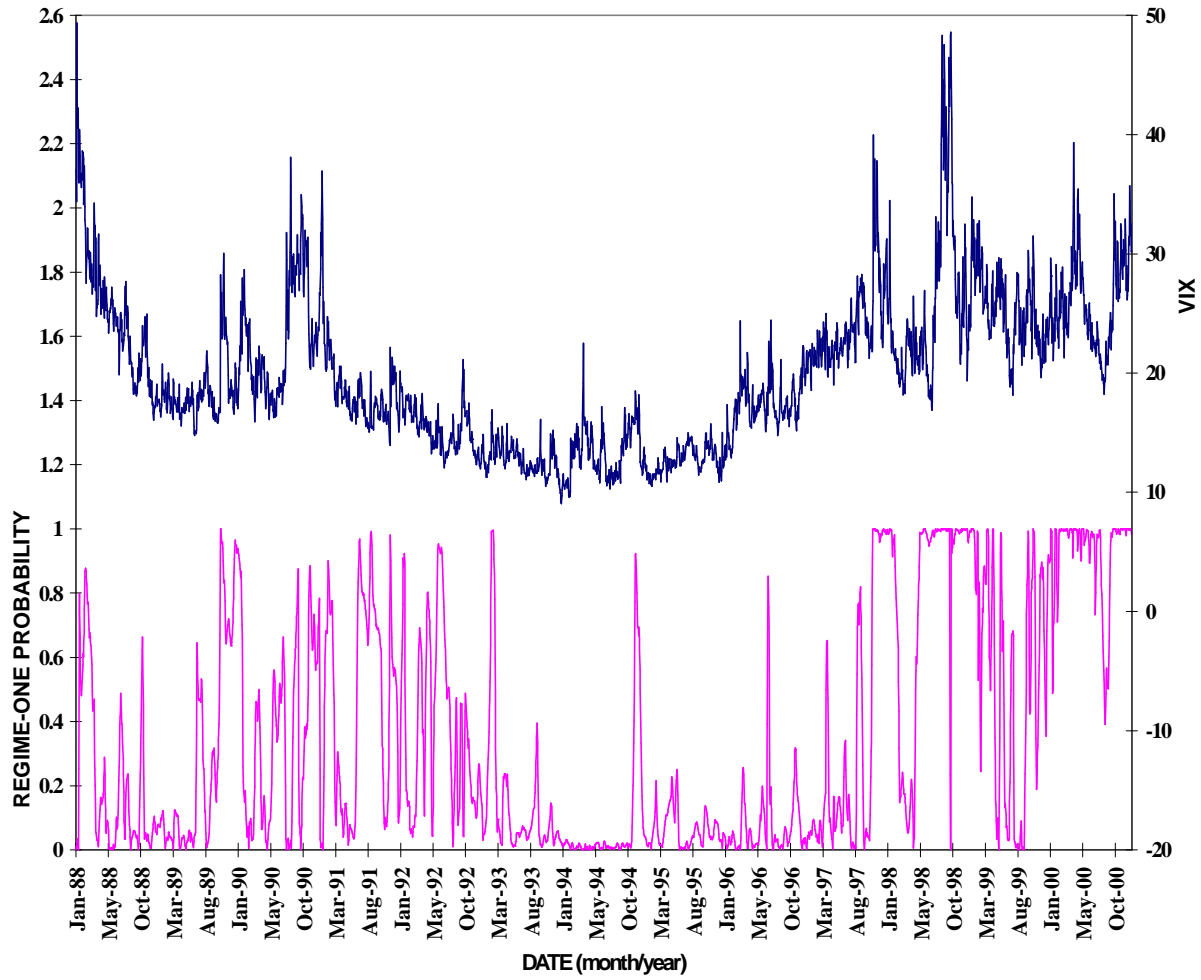


Figure 2

This figure displays the CBOE's Volatility Index (upper series) and the smooth probability of being in regime-one (lower series) from the regime-shifting model in Table 4 for the 30-year Treasury bond returns. The sample period is 1988 to 2000.

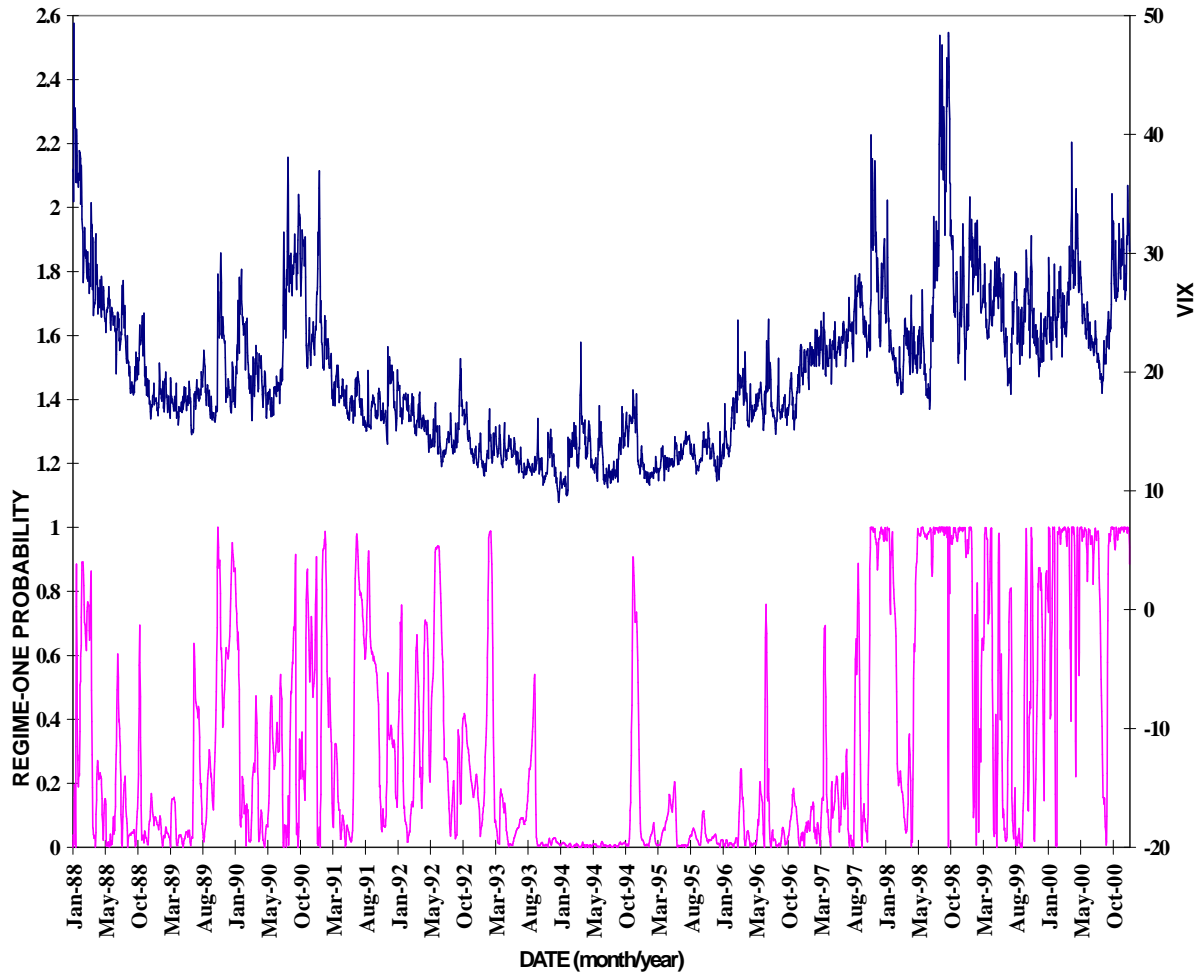
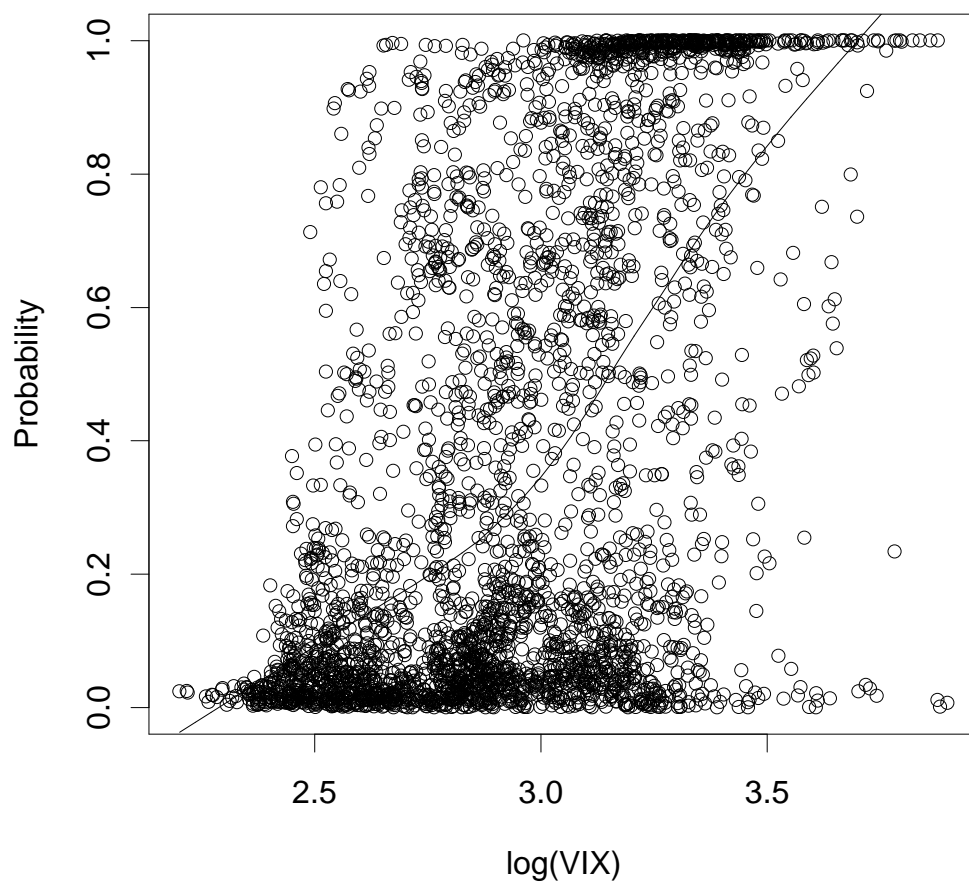


Figure 3

This figure plots the regime-one probability of 10-year T-bond against $\log(\text{VIX})$. The solid line is estimated nonparametrically using local polynomial fitting. The sample period is 1988 to 2000.



APPENDIX

Table A1: The relation between bond and stock returns and lagged VIX: Alternate Periods

This table reports how the relation between daily U.S. Treasury bond returns and stock returns varies with VIX_{t-2} for alternate sample periods. We estimate the following model:

$$B_t = a_0 + a_1 B_{t-1} + (a_2 + a_3 \ln(VIX_{t-2}))S_t + a_4 \ln(VIX_{t-2}) + \epsilon_t$$

where B_t and S_t are the daily T-bond and stock returns, respectively; $\ln(VIX_{t-2})$ is the natural log of the VIX in period $t - 2$; ϵ_t is the residual, and the a_i 's are estimated coefficients. The overall sample period is 1986 to 2000. The model is estimated by OLS and T-statistics are in parentheses, calculated with autocorrelation and heteroskedastic consistent standard errors per the Newey and West (1987) method with five lags. The last three rows report the total implied coefficient on the stock return at the median, 95th percentile, and 5th percentile of the lagged VIX value, respectively. Panel A reports results for the 10-year T-bonds and Panel B for the 30-year T-bonds.

| Panel A: 10-year U.S. Treasury Bonds | | | | |
|--|-------------------|--------------------|-------------------|--------------------|
| Sample Period | 1/88-12/00 | 1/86-12/00 | 1/86-6/93 | 7/93-12/00 |
| a_0 | -0.051 (-0.66) | -0.0826 (-0.94) | -0.04 (-0.23) | -0.183 (-1.92) |
| a_1 | 0.062 (3.69) | 0.0732 (3.40) | 0.0694 (2.49) | 0.0514 (2.23) |
| a_2 | 1.215 (8.81) | 0.762 (5.41) | 0.588 (5.04) | 1.757 (11.57) |
| a_3 | -0.351 (-8.17) | -0.201 (-4.73) | -0.134 (-4.24) | -0.525 (-11.13) |
| a_4 | 0.0234 (0.89) | 0.0338 (1.12) | 0.0211 (0.35) | 0.0661 (2.01) |
| R^2 (%) | 10.77 | 9.53 | 11.83 | 15.29 |
| $a_2 + a_3 \ln(VIX)$ (at the median VIX) | 0.188 | 0.166 | 0.195 | 0.175 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 95th percentile) | 0.010 | 0.068 | 0.123 | -0.050 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 5th percentile) | 0.354 | 0.266 | 0.243 | 0.485 |

Table A1: (continued)

| Panel B: 30-Year U.S. Treasury Bonds | | | | |
|--|-------------------|-------------------|-------------------|--------------------|
| Sample Period | 1/88-12/00 | 1/86-12/00 | 1/86-6/93 | 7/93-12/00 |
| a_0 | -0.089 (-0.77) | -0.147 (-1.11) | -0.125 (-0.48) | -0.27 (-1.92) |
| a_1 | 0.017 (1.02) | 0.0415 (2.12) | 0.047 (1.92) | 0.01 (0.43) |
| a_2 | 1.85 (9.14) | 1.199 (6.15) | 0.944 (5.70) | 2.69 (13.57) |
| a_3 | -0.526 (-8.31) | -0.312 (-5.29) | -0.213 (-4.67) | -0.796 (-13.09) |
| a_4 | 0.037 (0.93) | 0.056 (1.23) | 0.050 (0.56) | 0.097 (1.99) |
| R^2 (%) | 11.66 | 10.75 | 13.8 | 15.33 |
| $a_2 + a_3 \ln(VIX)$ (at the median VIX) | 0.309 | 0.274 | 0.319 | 0.288 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 95th percentile) | 0.042 | 0.123 | 0.204 | -0.052 |
| $a_2 + a_3 \ln(VIX)$ (at VIX's 5th percentile) | 0.558 | 0.430 | 0.394 | 0.758 |